Title: *Monitoring of delamination defects in composite beams*

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ABSTRACT

Damage detection systems based on array of piezoelectric transducers sending and receiving strain waves have been the subject of intensive research in the last decade. The signal processing problem is the major challenge in this concept and soft computing methods are the most often suggested tools for developing a numerically efficient solver.

Delamination in composite beams is an example of structural defect considered in this paper. The objective is to propose a new approach to solve the inverse dynamic problem of structural health monitoring. The first proposition provides an *a posteriori* analysis. It is based on the Virtual Distortion Method (VDM), using the concept of the dynamic influence matrix. The VDM method allows for decomposition of the dynamic structural response into components caused by external excitation in an undamaged structure (the linear part) and components describing perturbations caused by the internal defects in the structure (the nonlinear part). As a consequence, analytical formulae for calculation of these perturbations and the corresponding gradients can be derived. Assuming the inspection zone of all possible defects, a gradient-based optimization algorithm is applied to solve the problem of defect identification. A numerical tool for delamination identification in double-layered beams has been developed. An experimental verification of the concept has been carried out.

The *a posteriori* VDM-based approach detects and identifies already existing delaminations. However, sensor system mounted permanently on the operating structure can also be used for real-time health monitoring. Therefore the second proposition presented in the paper is the real-time detection system based on monitoring the strain evolution (measured by piezo-patches) due to the delamination defect. The advantages and shortcomings of both (*a posteriori* and real-time) approaches as well as the possibilities of their application are discussed. Numerical examples and experimental results are presented.

INTRODUCTION

Delamination is one of the most dangerous kinds of damage. Fast development of layered materials and composites in last years, is an incentive to look for effective identification methods. Some techniques based on signal processing have been proposed, recently. From the mathematical point of view, determination of damage location and size on the basis of structural response is an optimisation task. One of the most often suggested approach to develop numerically efficient solver for this task is soft computing e.g. neural networks, genetic alghoritms.

The crucial point for the identification tools is to have correct numerical model of delamination. In this paper a proposition of such model will be presented in the framework of the Virtual Distortion Method (VDM).

VDM FORMULATION

The Virtual Distortion Method allows for fast, efficient reanalysis due to stiffness modification in structural elements. It means that using information from a FEM model of the original structure, we are able to obtain structural response after modification very quickly as a solution of local problem. The most important definitions for VDM are:

- virtual distortion – a strain generated in an element modeling stiffness modification in the element (its influence on the structure can be considered as a result similar to the effect non-homogenous heating)
- influence matrix – collection of structural responses to unit virtual distortion imposed in every element of the structure successively.

Computation of the influence matrix, which can be realized by introducing compensative forces equivalent to unit virtual distortions, is the basis for VDM analysis. The current strain in an element is a superposition of the linear response (corresponding to undamaged structure) due to external load and the residual response (corresponding to damaged structure) due to parameter modification:

$$\varepsilon_i = \varepsilon_i^L + \varepsilon_i^R = \varepsilon_i^L + \sum_j D_{ij}\varepsilon_j^0 \tag{1}$$

The residual response is a linear combination of influence matrix components and virtual distortions as shown in (1).

The constitutive relation for the structure with introduced virtual distortions yields:

$$P_i = E_i A_i \left(\varepsilon_i - \varepsilon_i^0\right) \tag{2}$$

The internal forces P_i can be also expressed in terms of the modified structure parameters in the following way:

$$P_i = E_i^* A_i^* \tag{3}$$

where E_i, A_i – unmodified parameters, E_i^*, A_i^* - parameters after modification. By comparing (2) and (3), we obtain the formula for a modification coefficient:

$$\mu_i = \frac{E_i^* A_i^*}{E_i A_i} = \frac{\varepsilon_i - \varepsilon_i^0}{\varepsilon_i} \tag{4}$$

Using this coefficient, equation (1) can be expressed as:

$$\varepsilon_i = \varepsilon_i^L + \varepsilon_i^R = \varepsilon_i^L + (1-\mu_i)\sum_j D_{ij}\varepsilon_j \tag{5}$$

Equation (5) can be converted into simple matrix equation $A\varepsilon^0 = b$, where the virtual distortion vector is the only unknown.

Introducing the time dimension in VDM approach leads to the Impulse Virtual Distortion Method (IVDM) formulation, which allows to model dynamic response for modified structure. The fundamental equation (1) can be written as:

$$\varepsilon_i(t) = \varepsilon_i^L(t) + \varepsilon_i^R(t) = \varepsilon_i^L(t) + (1-\mu_i)\sum_{t=0}^{\tau}\sum_j D_{ij}(t-\tau)\varepsilon_j(\tau) \tag{6}$$

Influence matrix elements in this case are computed as structural elements responses to a time-dependent unit virtual distortion.

DELAMINATION MODELLING ALGHORITM

The VDM/IVDM have been applied for modeling of delamination in a double layered beam. The idea was to build a numerical model of a structure (Fig. 2) with a special, very thin layer between two beams. This extra layer, called in the paper contact layer, can be interpreted as a glue layer in a real structure.

In the modelling of contact layer the following conditions have been imposed on the pair of inclined elements (left and right) and on the vertical one in each delamination modelling cell "*i*" (cf. Fig.1):

- in the case of vertical compression in delamination cell "*i*":

$$\begin{gathered} \varepsilon_{iR}^0(t) = \varepsilon_{iR}(t) \quad \varepsilon_{iL}^0(t) = \varepsilon_{iL}(t) \\ \varepsilon_{iN}^0(t) = \varepsilon_{iN}(t) \end{gathered} \tag{7}$$

- in the case of non-compressive vertical interactions in delamination cell "*i*":

$$\begin{gathered} \varepsilon_{iR}^0(t) = \varepsilon_{iR}(t) \quad \varepsilon_{iL}^0(t) = \varepsilon_{iL}(t) \\ \varepsilon_{iN}^0(t) = 0 \end{gathered} \tag{8}$$

where:

$$\begin{gathered} \varepsilon_{iR}(t) = \varepsilon_{iR}^L(t) + \sum_{t=0}^{\tau}\sum_{j,k} D_{iR,jk}(t-\tau)\varepsilon_{jk}^0(\tau) \ ; \ \varepsilon_{iL}(t) = \varepsilon_{iL}^L(t) + \sum_{t=0}^{\tau}\sum_{j,k} D_{iL,jk}(t-\tau)\varepsilon_{jk}^0(\tau) \\ \varepsilon_{iN}(t) = \varepsilon_{iN}^L(t) + \sum_{t=0}^{\tau}\sum_{j,k} D_{iN,jk}(t-\tau)\varepsilon_{jk}^0(\tau) \ ; \ k = R, L, N \end{gathered} \tag{9}$$

and $D_{iR,jk}$, $D_{iL,jk}$, $D_{iN,jk}$ denote influence vectors describing strains generated in elements: right, left and normal in the cell "*i*" induced due to unit virtual distortion generated in element "*jk*".

The above conditions lead to the following effects modelling the contact problem in the gap generated due to delamination:
- in the case of vertical compression in delamination cell "*i*" the shear forces vanish in the contact layer,
- in the case of non-compressive vertical interactions in delamination cell "*i*" the shear forces as well as the verticall forces vanish.

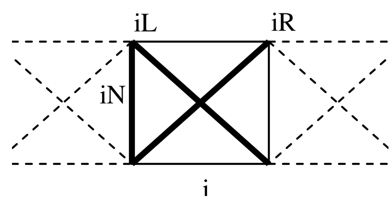
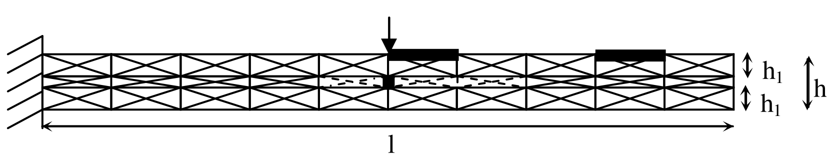


Fig.1 Notation used in description of the contact layer.

The sets of equations (7), (8) allow for determination of virtual distortions modelling shear movement and transversal gap development along the contact layer. The resultant strains in contact elements take the form (9).The formulas for delamination modelling was presented here for dynamic case. For static case the same equations hold but there is no summation over time.

NUMERICAL EXAMPLE AND EXPERIMENTAL VERIFICATION

In case of extensive delamination, the efect of an open gap can appear, when the normal forces in the contact layer (in delamination zone) vanish. Such situation is illustrated in Fig. 4.



l=0.5 m h_1=0.005 m h=0.0101 m	material properties for beams: E=140e9 Pa , ρ=7800 kg/m^3 material properties for inner layer: E=0.1e8 Pa , ρ=1 kg/m^3

Fig.2 Numerical model (dotted lines damage, bold lines sensors, arrow excitation point)

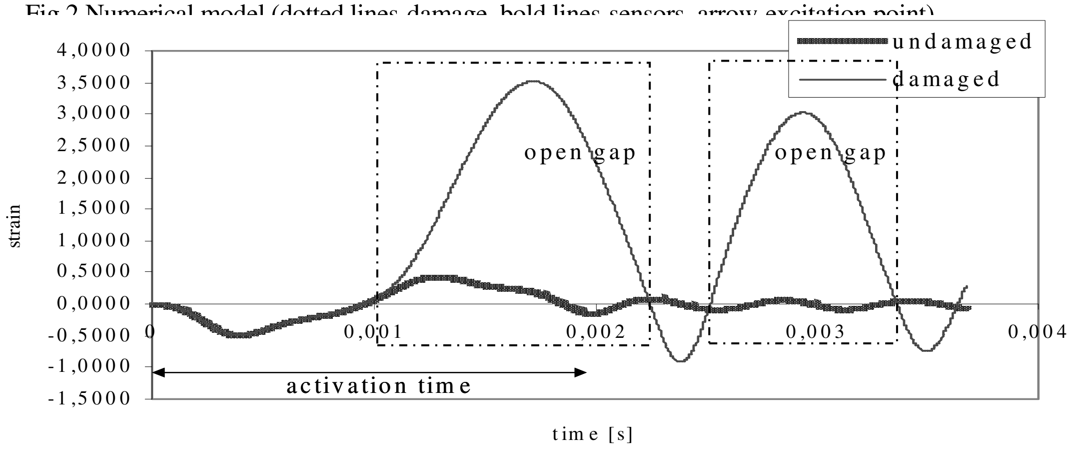
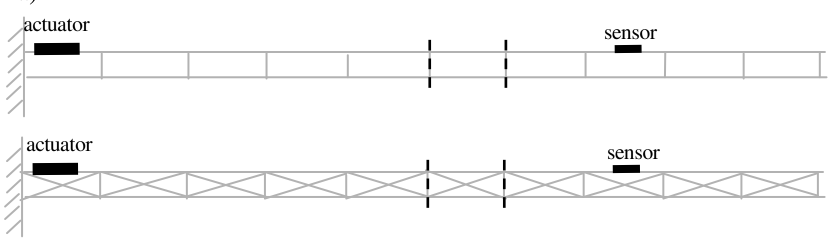


Fig.3 Extensive delamination case - responses of the undamaged and damaged structure for vertical element localized inside the delaminated zone.

The damage was localized in the middle part of the structure (elements marked as dotted lines) and delamination crack size was near 1/3 length of the beam.
As it is shown in Fig. 4c, when the strains for a vertical element placed in the damaged region (bold line) become positive, virtual distortions β_i^0 take non-zero values (it's the open gap situation).

The experimental verification of the delamination modeling alghoritm was done for a simple structure built from two beams connected in ten points by screws. The structure was excited by a windowed signal, which was applied to the piezoelectric actuator mounted near the clamped end. Structural response was collected using piezoelectric patch glued near the free end. The results are presented in Fig. 4.

a)



b)

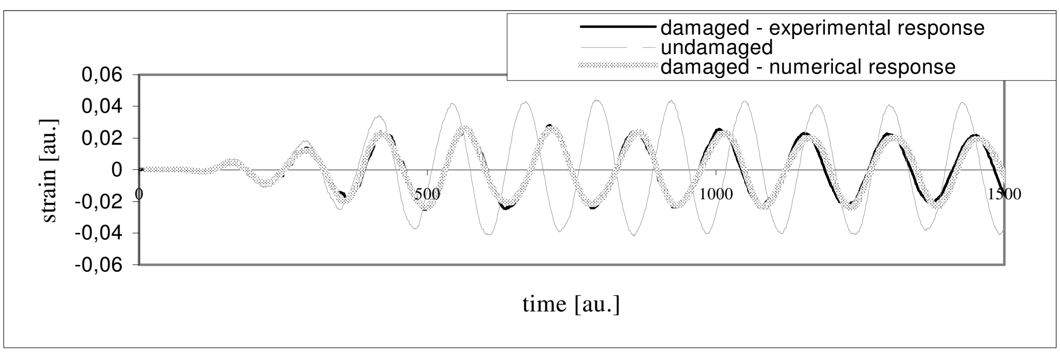


Fig.4 Experimental verification results: real structure with vertical lines as screws and numerical model with contact layer (dotted lines correspond damaged screws) (a), experimental and numerical response comparison (b).

VDM APPROACH IN DELAMINATION MONITORING

We shall pose the optimisation problem of structural damage identification (constraining ourselves temporarily to the static case) within the framework of the Virtual Distortion Method (cf. [4]). Let us minimise the following function:

$$\min \sum_{A} \left(\varepsilon_A^M - \varepsilon_A\right)^2 \tag{10}$$

which can be interpreted as an average departure of the total structural strain ε_A from the in-situ measured strain ε_A^M in damaged locations *A*. Taking advantage of the VDM formulation we can decompose the strain ε_A into two parts:

$$\varepsilon_A = \varepsilon_A^L + \varepsilon_A^R = \varepsilon_A^L + \sum_A D_{Ai}\varepsilon_i^o \tag{11}$$

As the component ε_A^L is constant for a given external load, the so-called residual strain component ε_A^R may only be varying in the optimisation process with the virtual distortion ε^o as the design variable.

We shall measure the structural damage in each member *i* with the help of the coefficient μ_i i.e. with the ratio of cross-sectional areas of a damaged member to the undamaged one. Consequently we have to impose appropriate constraints on this coefficient. As we examine the physical process of deterioration of the member cross-section we are interested in such μ_i, which complies with the following constraints:

$$0 \le \mu_i \le 1 \quad \text{i.e.} \quad 0 \le \frac{\varepsilon_i - \varepsilon_i^o}{\varepsilon_i} \le 1 \tag{12}$$

For delamination problems the coefficient μ will finally (after optimisation) take only two values: 0 (delamination) or 1 (full connection).

The gradients of the objective function and the constraints are expressed in terms of the design variable ε^o as follows:

$$\nabla f = \frac{\partial f}{\partial \varepsilon_k^o} = -2\sum_A D_{Ak}\left(\varepsilon_A^M - \varepsilon_A\right) \tag{13}$$

and

$$N = n_{kl} = \frac{\partial g_l}{\partial \varepsilon_k^o} = \frac{\delta_{lk}\varepsilon_l - D_{lk}\varepsilon_l^o}{\left(\varepsilon_l\right)^2} \tag{14}$$

In order to solve the damage identification problem posed by (10) and (12) the Gradient Projection Method (cf. [1], [2], [3]) can be used as optimisation tool. The Gradient Projection Method is based on the idea of projecting the search direction (i.e. the direction in which the objective function value decreases) into the subspace tangent to the active constraints.

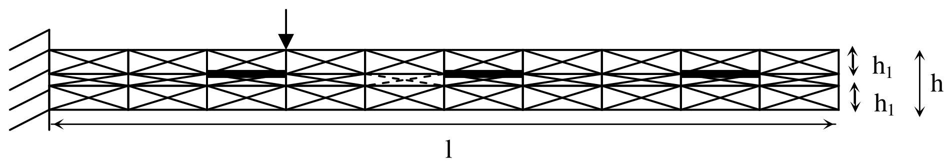
NUMERICAL TESTS

Let's discuss the case of damage located in section 5 (Fig. 5). The excitation signal is one period of the sinusoidal wave of the frequency corresponding to 4th natural frequency of the undamaged structure (666.16 Hz). The initial gradient values are very important for the optimization routine. For the one-section delamination damage case, the gradient disturbance has a localized character. It means that optimal sensor location is near the damaged section (initial gradients for sensors placed long distance from damage are flat).

There will be two cases of sensor location compared. In the case of one sensor placed near the free end of the structure, gradient values at the begining of the optimization process are at the same level (do not show that damage is localized in section 5). After 80 iterations the goal function value is approaching zero but still decreasing. As shown in Fig. 6a the damage coefficient for section 5 is also near zero. It seems to be promising for identification but very time-consuming.

In the case of three sensors the first gradient with index 5 is at higher level than others, and we have quite good identification results already after 20 iterations (Fig. 6b).

The computation of gradients for this case was done for every of the three sensors (in every optimization iteration). The way for doing computation faster is to evaluate the most possible location of damage (using results from a few sensors) and compute gradients only for the sensor with the biggest difference between the damaged and undamaged response.



l=0.5 m
h_1=0.005 m
h=0.0101 m

material properties for beams:
E=140e9 Pa , ρ=7800 kg/m^3
material properties for inner layer:
E=140e9 Pa , ρ=1 kg/m^3

Fig.5 Truss-structure model (not scaled) with the sensors (bold lines) and the excitation point (arrow) .

a)

b)

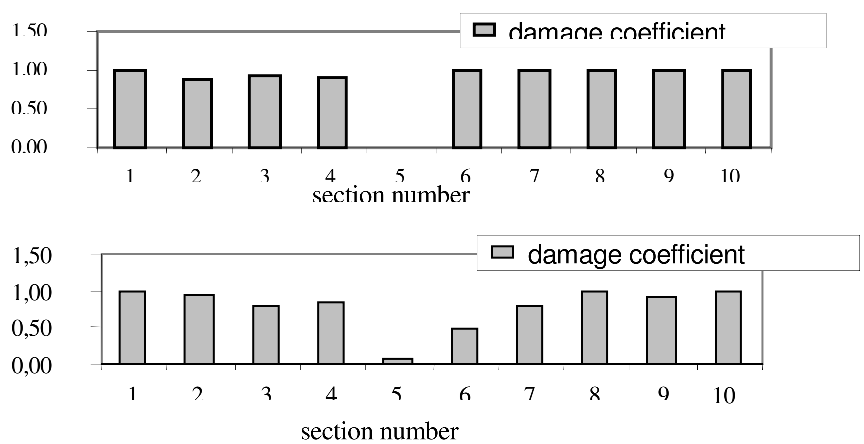


Fig. 6 Identification results for one sensor (a) and for three sensors (b).

REAL-TIME DETECTION OF DELAMINATION

The previously discussed cases can be considered as *a posteriori* testing detecting and identifying already existing delaminations. However, in many applications a real-time health monitoring is the challenge. Having sensor system mounted permanently on the operating structure, the damage detection “in motion” seems to be feasible. Let us discuss the case of our testing beam under steady-state excitation with delemination (in section No.5) generated during exploitation.

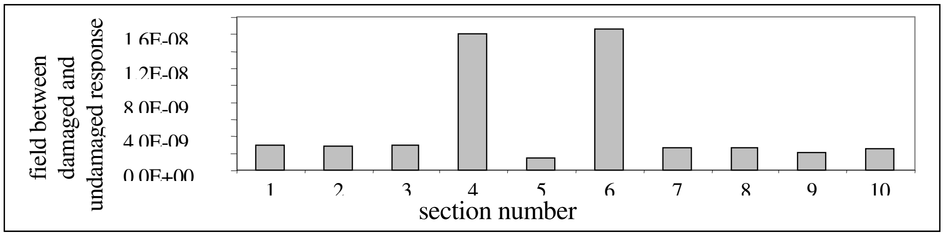


Fig.7 Field between damaged and undamaged response for delamination in section 5.

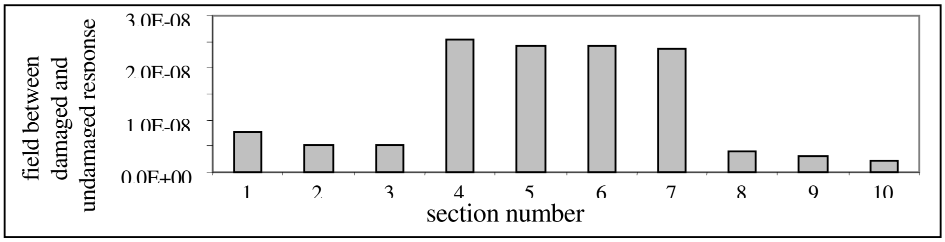


Fig.8 Field between damaged and undamaged response for delamination in sections 5 and 6.

The field between standard response on steady-state excitation and response changed by delamination computed for each sensor are shown in Fig. 7, 8.

Alghoritm for automatic identification of delamination based on the above results can be proposed. The algorithm can be based on determination of the pair of two sensors with maximal average signal. The delamination area is located in-between these maximally loaded sensors. The example with more extended delamination (in elements No. 5 and 6) is shown in Fig. 8, where locations of maximally loaded sensors determine the length of the defect.

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